Phys 402 Fall 2022 Homework 5

Due Wednesday, 5 October @ 10 AM as a PDF upload to ELMS

1. Griffiths and Schroeter *Quantum Mechanics*, 3^{rd} Ed., Problem 7.52 (Degenerate perturbation theory for a two-dimensional charged harmonic oscillator in a magnetic field. *Hint: To solve the TISE try separation of variables:* $\psi(x, y) = X(x)Y(y)$. *Also, make use of Section 7.2.1 – Two-fold Degeneracy, starting on page 286 of Griffiths.*)

2. Estimate the size of the internal magnetic field in the Hydrogen atom. Start with Griffiths Eq. [7.60] and use the Bohr radius a as an estimate of r, and \hbar as an estimate for $|\vec{L}|$. Express your answer in Tesla and compare to the earth's magnetic field which is about 50 μ T.

- **3**. Griffiths and Schroeter *Quantum Mechanics*, 3^{rd} Ed., Problem 7.24 (*Weak* Zeeman splitting for the H-atom n = 2 states)
- 4. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 7.27 (*Strong* Zeeman effect for the H-atom n = 2 states)
- Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 7.47 (Hyperfine splitting in Deuterium)

6. Consider the Schrodinger equation for the un-perturbed Helium atom, which is that of the two electrons independently orbiting the same nucleus:

$$\mathbf{H}^{0} = -\frac{\hbar^{2}}{2m}\nabla_{1}^{2} + \frac{(-e)(+2e)}{4\pi\varepsilon_{0}r_{1}} - \frac{\hbar^{2}}{2m}\nabla_{2}^{2} + \frac{(-e)(+2e)}{4\pi\varepsilon_{0}r_{2}}$$

where the Laplacian operators only operate on the spherical coordinates of either electron 1 or electron 2. Note that the electrons are identical, they have exactly the same mass and charge. The solution to the un-perturbed Schrödinger equation is a wavefunction that depends on 6 coordinates:

$$\mathrm{H}^{0}\Psi^{0} = E_{T}\Psi^{0}$$

where $\Psi^0 = \Psi^0(r_1, \theta_1, \phi_1, r_2, \theta_2, \phi_2)$. To proceed, we try an ansatz which we believe will lead to separation of variables as $\Psi^0(r_1, \theta_1, \phi_1, r_2, \theta_2, \phi_2) = \psi_a(1)\psi_b(2)$ where "1" and "2" represent all of the coordinates of electrons 1 and 2, respectively, and "*a*" and "*b*" are different lists of quantum numbers, in general. Show that this leads to two Schrodinger equations, one for each electron, with the constraint that $E_T = E_a + E_b$.

EXTRA CREDIT

5. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 5.35 (Degeneracy pressure in white dwarfs)

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"At this point we notice that this equation is beautifully simplified if we assume that space-time has 92 dimensions."